Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
A		2207464

Victory or Defeat: Rider Power

Summary

Nowadays, outdoor sports are forming a trend around the world. In particular, cycling individual time trial has become an important event in large-scale sports competitions. To minimize the total game time, we formulate the riding strategy scientifically and rationally for cyclists in this paper.

To begin with, we divide the track into reasonable segments: smooth road, uphill, downhill and bend. From the laws of kinematics and dynamics in physics, we can determine the basic handling rules of the rider in different positions on the road, such as : for flat road, drive at a constant speed; for uphill, accelerate in advance and decelerate during uphill; for long and gentle downhill, you need to apply traction at the top of the slope to make it slide down at a constant speed ; for short and steep downhills, it can naturally accelerate and slide down; for corners, naturally decelerate when entering the corner, and accelerate quickly when exiting the corner. Based on the above rules, we construct a rider power-position dynamics model. In the meanwhile, we set the goal of the shortest total time of the game and limit the total energy that the rider can consume and not to overload for a long time and establish an optimization model based on differential equations.

Then, we apply the above model to different types of riders and different time trial courses. We collect the original recorded data in literature. After data pre-processing, we finally obtain data on the maximum power that 56 males and 48 females could sustain over various lengths of time. Based on this data, we can draw power profiles for different types of riders. Besides, we apply the model according to the focus of the competition in different scenarios. For the 2021 Olympic Time Trial course in Tokyo, Japan, we focused on flat and ramp terrain. Likewise, for the 2021 UCI World Championship time trial course in Flanders, Belgium, we focus on flat and sharp-turn terrain. Moreover, we select the track in Brand Park, California as the designed track with a combination of the above two tracks. In the end, we can conclude that the maximum power output is required for uphill, the lower power is required for downhill, and the speed is reduced as little as possible for sharp turns.

Next, we perform sensitivity analyses on the above model. Further, we find that the total duration of the race is insensitive to different wind directions, small changes in wind strength, and small deviations from target power. We conclude that the model has passed the sensitivity test.

Besides, to consider the optimal power use for a team time trial, our analysis shows that the team consists of two windbreakers and four sprinters, and the windbreakers lead the entire team to complete the task of the competition. Therefore, we use the above model for the windbreakers, and then obtain the optimal power distribution scheme for the windbreakers.

Finally, we discuss the strengths, weakness and further discussion of the model and offer rider's race guidance for a Directeur Sportif of a team.

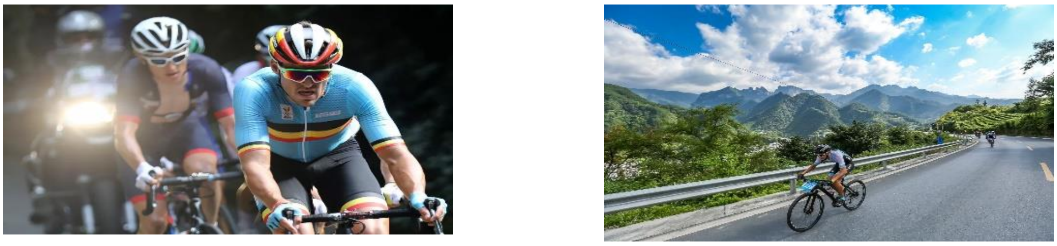
Keywords: power dynamics model, differential equation, cyclist, pattern recognition

Contents

1 Introduction

1.1 Problem Background

In recent years, with the vigorous development of the global fitness movement, outdoor sports projects close to nature have developed rapidly, and outdoor sports are forming a trend around the world. As an outdoor sport, road cycling is becoming more and more popular. Bicycle road races are a sport that tests the speed and endurance of competitors. Players of various types can not only enjoy the joy of riding, but also appreciate the beautiful scenery along the way. Among them, Bicycle road races include a criterium, a team time trial, and an individual time trial. As an important part of bicycle road races, individual time trial is deeply loved by cycling enthusiasts, and has become an important event in large-scale sports competitions such as the Olympic Games. In the competition, how to formulate the riding strategy scientifically and rationally is an important task of the Directeur Sportif, and it is also the key to the victory of the contestants.



(a) A team time trial (b) Individual time trial

Figure 1: Bicycle road races

(a) A team time trial: TTL is one of the cycling events. Each team of 4 athletes participates in the competition. When reaching the finish line, the time when the third athlete of the team arrives is the team's score. It was listed as a competition event in 1912.
Image source: https://www.jj20.com/bz/tyyd/ydxz/271985_2.html

(a) Individual time trial: ITT are a form of cycling competition where riders race against the clock. Leave the starting point in a staggered fashion. The key to winning is the rider's endurance and strength.
Image source: http://news.cnhubei.com/content/2021-10/11/cotent_14160041.html

1.2 Restatement of the Problem

Considering the background information on TTL and ITT, and the constraints stated in the question we should address the following problems:

- **Problem 1**: Build a mathematical model that can be applied to any type of rider to determine the relationship between the cyclist's position on the track and the power the rider applies. It's worth noting that there is a limit to the total energy a rider can expend throughout the process, as well as a limit to what was previously expended and accumulated over the power curve.
- **Problem 2**: Define power curves for two types of cyclists. One is a time trial specialist, and the other is a different type of player. In addition, the profiles of players of different genders should also be considered.

- **Problem 3**: Apply the model to various individual time trial scenarios. Each of the power profiles you defined above is listed below:

  ○2021 Olympic Time Trial course in Tokyo, Japan,

  ○2021 UCI World Championship time trial course in Flanders, Belgium,

  ○At least one track of your own design, including at least four sharp turns and one non-trivial road grade. The end of the course should be close to its beginning.
- **Problem 4**: The effects of weather conditions, including wind direction and strength, need to be determined, and how sensitive the model results are to weather and the environment.
- **Problem 5**: Since the rider is not necessarily following exactly the racing strategy set by the Directeur Sportif, it is possible to deviate from the target power. Therefore, knowledge of the possible range for short periods of time in critical sections of a given route is required to determine how sensitive the results are to the rider's deviation from target power.
- **Problem 6**: Discuss how to extend the model for optimal energy use in team time trials that include six riders per team. The race result is determined by the time the fourth rider crosses the finish line.

1.3 Literature Review

The importance of developing a competition strategy for bicycle road races in sports competition is widely recognized. Numerous theoretical studies have been conducted on individual time trial and team time trial strategies. By reviewing the literature, we found that the mainstream analysis methods are based on the following aspects [1-4]:

Wu et al. (2013) proposed that when formulating a cycling strategy, it is necessary to combine the individual physical fitness of the cyclists and comprehensively consider the changes of the track scene [1]. Ding et al (2018) analyzed the force of the bicycle in this process based on the turning situation in the cycling event [2]. Zhang et al. (2005) analyzed the principle of force in the process of bicycle driving. And the rolling friction force and static friction force received by the wheel during the running of the bicycle are analyzed emphatically [3]. Furthermore, according to Zhang et al. et al., we found that riders have different power curves in different road positions. Therefore, when building the model, we paid close attention to the impact of this aspect. Peter et al. (2021) analyzed critical power in cycling power training. The power curve is expounded in detail, and then the method of power analysis is outlined. Combined with the existing data in the laboratory, the field test was carried out, and the relationship between power and time in cycling was discussed [4]. The data part of it will also be used in our model.

1.4 Our work

The question asked us to analyze the relationship between the position of individual cyclists on the track and the power they used, and to help Directeur Sportif and the competitors to formulate a competition strategy. Our work mainly includes the following:

1) Dialectically considering the road conditions at different positions on the track, the relationship between power and position was modeled for men and women respectively.

2) Combined with pattern recognition technology, the real track is digitally processed, and the route is rasterized, so that the model is well combined with the actual scene.
3) Sensitivity analysis was carried out based on the influence of natural environment such as weather and track, and based on the models of power position dynamics model.
4) Expanded the model to a team of six players and gave rationalization suggestions.

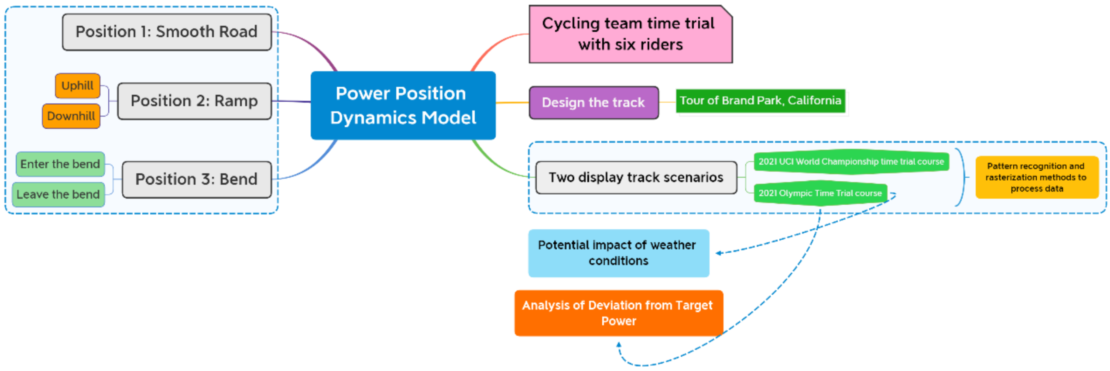


Figure 2: Track Analysis Chart

2 Assumptions and Justifications

➢ **Assumption 1: We assume that the bicycle is turning on a flat section.**

Justification: From Ding Danhua's research, it can be known that the turning of bicycles on the curve is divided into the turning on the slope and the turning on the plane. Considering that in the actual scene competition, especially in the two competitions in the requirements, the inclination angle of the slope involved in the turning is very small, so for the turning situation, this paper only considers the bicycle turning on a flat road section.

➢ **Assumption 2: We assume that the aerodynamic drag coefficient of the bicycle is constant throughout the track.**

Justification: From a physical knowledge point of view, the air resistance coefficient is related to the windward area of the object, the smoothness of the object and the overall shape factor. Since these factors are constant for the bike and rider throughout the race, we assume that the drag coefficient remains constant throughout the race.

➢ **Assumption 3: We assume that the traction force of the bicycle in the speed change movement of the race is constant force.**

Justification: When the bicycle performs shifting motions on slopes and curves, if the shifting motion is performed with a constant amount of force, the rhythm and frequency of driving can be better controlled, and the competition can be more efficient. An-other advantage of constant force over variable force processing is that during shifting movements, the calculation of the energy consumed by the rider can be simpler, and only need to deal with the changing distance.

➢ **Assumption 4: We assume that if we ignore the sprint case, the cyclist is driving at a constant speed on the smooth road.**

Justification: For the case where the bike is traveling on flat ground, we assume the bike is traveling at a constant speed. Ignoring the situation of sprinting, driving at a constant speed on a flat ground can ensure a relatively stable output of the rider's breathing and applied force, which is conducive to the change of the force of the back downhill, entering and exiting corners. The energy consumed when driving at a constant speed.

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
v_1	the speed of the bicycle on the smooth road	m/s
F_1	the force exerted by the ride	N
f_1	he rolling friction of a bicycle on the smooth road	N
k_1	the coefficient of air resistance	–
k_2	the coefficient of rolling friction	–
R_w	the radius of the bicycle wheel	cm
R_c	the radius of curvature of the curve at the inner point of the tangent	m
t_1	all time on level ground	s
t_2	the pre-uphill time and the uphill time	s

4 Power Position Dynamics Model

Considering that there are different positional sections in the individual time-trial cycle of the bicycle, and for different positional sections, the force and energy consumption exerted by the rider are different. Bicycle individual time trial sections can be roughly divided into three types: flat road, curve, and ramp. Therefore, this paper decomposes the total travel time into flat road time, curve time and ramp time. Discuss the power and energy expenditure exerted by the rider and the relationship between them.

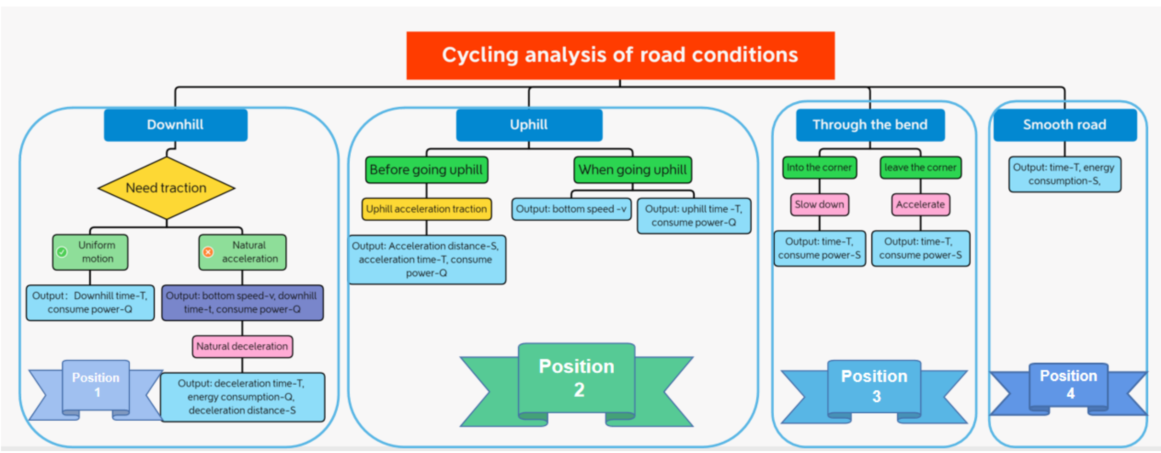


Figure 3: Track Analysis Chart

4.1 Model Building

4.1.1 The Position 1: Smooth Road

For the situation where the ground is always flat on a smooth road, we may wish to assume that the rider is driving at a constant speed on the smooth ground, and the applied force is equal to all frictional resistance, then there is:

$$F_1 = f \tag{1}$$

$$f = f_1 + k_1 v_1 \tag{2}$$

Where, F_1 indicates the force exerted by the ride, f represents all frictional resistance f_1. indicates the rolling friction of a bicycle on the smooth road, k_1 indicates the coefficient of air resistance, v_1 indicates the speed of the bicycle on the smooth road.

According to the calculation formula of rolling friction force, then there is:

$$f_1 = \frac{(m_1 + m_2)g \cdot k_2}{R_w} \tag{3}$$

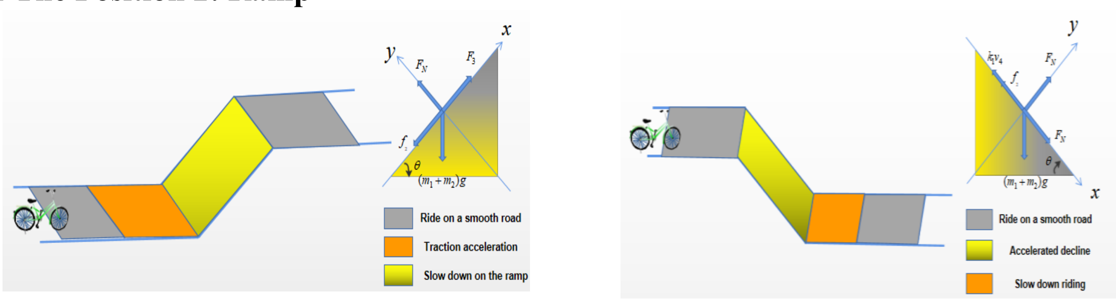
Where, m_1 indicates quality of rider, m_2 indicates quality of bicycle, k_2 indicates the coefficient of rolling friction, R_w indicates the radius of the bicycle wheel.

Therefore, the energy consumed by the rider on flat ground is:

$$Q_1 = F_1 v_1 t_1 \tag{4}$$

Where, t_1 indicates all rider's time on level ground.

4.1.2 The Position 2: Ramp



(a) Go up the ramp (b) After Going Ramp

Figure 4: Pass the ramp

Step1: Before Going Ramp

According to the knowledge of dynamics in physics, the process of going ramp is a deceleration movement, so before going ramp, there will be a section of plane journey that is accelerating movement, until it reaches the bottom of the slope, the speed reaches the maximum.

We assume that the acceleration provided by the rider before going ramp is a constant force, the acceleration of the bicycle is

$$a_1 = \frac{F_2 - f_1 - k_1 v_2}{m_1 + m_2} \tag{5}$$

Where, F_2 indicates the force exerted by the ride, v_2 indicates the speed of the bicycle before going ramp.

According to the acceleration definition expression, we infer to:

$$\begin{cases} a_1 = \dfrac{F_2 - f_1 - k_1 v_2}{m_1 + m_2} = \dfrac{dv_2}{dt} \\ v|_{t=0} = v_1 \end{cases} \tag{6}$$

From equation 6, it can be known that the function relationship of the speed before the ramp with respect to time can be solved:

$$v_2 = v_2(t) \tag{7}$$

Then, the function relationship between time and speed before uphill is obtained by inverse solution:

$$t = t(v_2) \tag{8}$$

According to equation 10, the speed at the bottom of the slope can be obtained, and then from the speed on the smooth road, the acceleration before going up the slope can be obtained:

$$\Delta t_{11} = t(v_b) - t(v_1) \tag{9}$$

Where, Δt_1 indicates acceleration time before going ramp.

The distance traveled by the bicycle in the Δt time period is:

$$x = \int_{\Delta t_1} v_2 dt \tag{10}$$

Therefore, the energy consumed by the rider before going ramp is

$$Q_{21} = F_2 x \tag{11}$$

Step2: When Going Ramp

For the deceleration movement in the uphill, the speed of reaching the bottom of the Ramp before the Ramp can be determined according to the law of conservation of energy.

$$F_3 S + \frac{1}{2}(m_1 + m_2)\left(V_b^2 - V_1^2\right) = (m_1 + m_2)gh + W_r \tag{12}$$

Where, F_3 indicates the force exerted by the ride on the ramp, S indicates the length of ramp, h indicates the height of the ramp, v_b indicates the speed at which the bike reaches the bottom of the ramp, W_r indicates work done by all resistance.

From the expression for work done by resistance, we can get:

$$W_r = f_2 S + W_a \tag{13}$$

Where, f_2 indicates the rolling friction of a bicycle on the ramp, W_r indicates work done by all resistance.

According to the calculation formula of rolling friction force, then there is:

$$f_2 = \frac{(m_1 + m_2)g\cos\,\theta \cdot k_2}{R_w} \tag{14}$$

Where, θ indicates the ramp angle.

According to the acceleration definition expression, we get:

$$\begin{cases} a_2 = \dfrac{F_3 - J_2 - k_1 v_3 - (m_1 + m_2)g\sin\,\theta}{m_1 + m_2} = \dfrac{dv_3}{dt} \\ v_3|_{t=0} = v_b \end{cases} \tag{15}$$

From equation 15, it can be known that the function relationship of the speed on the ramp with respect to time can be solved:

$$v_3 = v_3(t) \tag{16}$$

It is easy to calculate that the bicycle uphill time is:

$$\Delta t_{12} = t(v_1) - t(v_b) \tag{17}$$

Similar to the previous analysis of the preparation for the ramp situation, We can get the distance s_1 as a function of time during the uphill deceleration motion.

$$s_1 = s_1(t) \tag{18}$$

Obviously, the distance is a monotone function with respect to time, so using the inverse function, the function of time with respect to the distance can be inversely solved:

$$t = t(s_1) \tag{19}$$

Substitute equation 18 into equation 16, we can get the relationship between the speed of the bike on the ramp and the distance traveled.

$$v_3 = v_3(t(s_1)) \tag{20}$$

Therefore, we can get the work done by the air resistance during the ramp process as:

$$W_a = k_1 \int_s v_3 ds_1 \tag{21}$$

Besides, the energy consumed by the rider on the ramp is:

$$Q_{22} = F_3 S \tag{22}$$

To simplify the notation, we denote the pre-uphill time and the uphill time as:

$$t_2 = \Delta t_{11} + \Delta t_{12} \tag{23}$$

Step3: After Going Ramp

For the downhill situation, we assume that the rider does not provide any power when going downhill, and let the rider and bike together perform a natural acceleration motion until the bottom of the slope is reached. The rider then reduces the overall speed to the speed of the smooth road in position 1 through a process of natural deceleration movement on flat ground.

[I] Short and Steep Ramp

- **Before Reaching the Bottom of the Ramp**

Before reaching the bottom of the slope, for short and steep slopes, the slope angle θ is large, and the bicycle and the person can be guaranteed to accelerate without the action of traction, and the acceleration can be expressed as:

$$\begin{cases} a_3 = \dfrac{(m_1 + m_2)g\sin\theta - f_2 - k_1 v_4}{m_1 + m_2} = \dfrac{dv_4}{dt} \\ v_4|_{t=0} = v_1 \end{cases} \tag{24}$$

Where, v_4 indicates the speed of the bicycle before reaching the bottom of the ramp.

Similar to the discussion of the ramp situation, the time until the downhill reaches the bottom of the ramp can be expressed as:

$$\Delta t_2 = t(v_b') - t(v_1) \tag{25}$$

Where, v_b' indicates the speed of the bicycle when reaching the bottom of the ramp.

- **After Reaching the Bottom of the Ramp**

After reaching the bottom of the ramp, the bicycle and the rider as a whole still do a natural deceleration motion until the speed is reduced to v_1 It is easy to know that the overall motion acceleration of the bicycle and the rider can be expressed as

$$\begin{cases} a_4 = \dfrac{-f_1 - k_1 v_5}{m_1 + m_2} = \dfrac{dv_5}{dt} \\ v_5|_{t=0} = v_h \end{cases} \tag{26}$$

Where, v_5 indicates the speed of the bicycle after reaching the bottom of the ramp.

Similar to the discussion of the ramp situation, the time it takes for the bicycle and the person as a whole to reduce their speed to v_1 after passing the bottom of the slope is:

$$\Delta t_3 = t'(v_b') - t'(v_1) \tag{27}$$

Where, v_b' indicates the speed of the bicycle when reaching the bottom of the ramp.

In particular, the bike and the person as a whole do the entire downhill movement, and the rider does not consume any power, so there is:

$$Q_{23} = 0 \tag{28}$$

To simplify the notation, we record the downhill time and the rider's energy expended as

$$\begin{gathered} t_3 = \Delta t_2 + \Delta t_3 \\ Q_2 = Q_{21} + Q_{22} + Q_{23} \end{gathered} \tag{29}$$

[II] Long and Gentle Ramp

- **Before Reaching the Bottom of the Ramp**

Before reaching the bottom of the slope, for long and gentle slopes, for long and gentle slopes, the slope angle θ is relatively small. If there is no traction during the downhill process, the bicycle and the person as a whole may decelerate or even stop on the slope when the speed is reduced to 0. therefore. We consider the position at the top of the slope.

$$\begin{gathered} F_3' + (m_1 + m_2)g\sin\,\theta - f_2 - k_1 v_1 = 0 \\ \Rightarrow F_3' = f_2 + k_1 v_1 - (m_1 + m_2)g\sin\,\theta \end{gathered} \tag{30}$$

Where, F_3' indicates the traction exerted by the rider during the descent. This size of traction can ensure that the bicycle reaches the bottom of the slope at a constant speed, that is, it reduces the time for the back to recover the original speed due to downhill acceleration.

Similar to the discussion of the ramp situation, the time until the downhill reaches the bottom of the ramp can be expressed as:

$$\Delta t_2' = \frac{s_2}{v_1} \tag{31}$$

Where, the speed of the bicycle to the bottom of the slope is still v_1.

- **After Reaching the Bottom of the Ramp**

After reaching the bottom of the ramp, because the speed at the bottom of the slope is still the speed v on the flat ground, it returns to the situation of the flat road in position 1.

Especially, before reaching the bottom of the slope, the energy consumed by the rider is:

$$Q_{23}^{'} = F_3^{'} s_2 \tag{32}$$

Where, s_2 indicates length of downhill.

To simplify the notation, we record the downhill time and the rider's energy expended as:

$$\begin{gathered} t_3' = \Delta t_2' \\ Q_2' = Q_{21} + Q_{22} + Q_{23}' \end{gathered} \tag{33}$$

4.1.3 The Position 3: Bend

For the case of corners, It can be seen that the bicycle and the person as a whole pass through the curve into two processes: from the entry point of the gentle straight to the inner point of the curve [5], and then from the inner point of the curve to the exit point, the speed recovers to a gentle level straight speed v_1 .

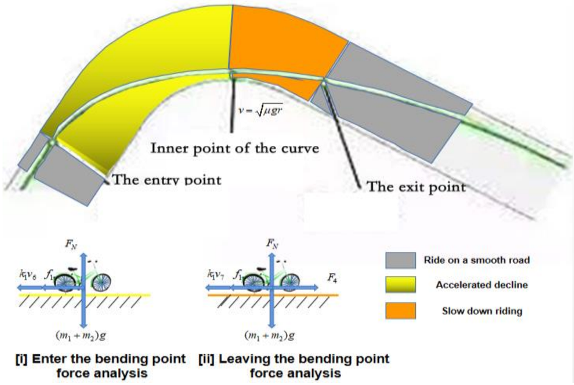


Figure 5: Go through the bend

Step1：The Entry Point Enters the Inner Point of the Curve

During the process from the entry point to the inner point of the curve, the bicycle and the person as a whole decelerate under the action of frictional resistance. At this time, the tangential acceleration is:

$$a_5 = \frac{-f_1 - k_1 v_6}{m_1 + m_2} = \frac{dv_6}{dt} \tag{34}$$

Where, v_6 indicates the speed of the bicycle from the entry point to the inner point of the curve.

For the selection of the inner point of the tangent and bending, according to the curvature calculation formula of the curve, we choose the point with the largest curvature of the curve arc as the tangent inner point of the curve.

$$K = \lim_{\Delta s \to 0} \left| \frac{\Delta \alpha}{\Delta s} \right| \tag{35}$$

Where, $\Delta\alpha$ indicates small arc tangent corner, Δs indicates micro arc length.

For the calculation of the speed at the inner point of the cut corner, we can know from the principle of the source of the centripetal force when the bicycle is turning on the plane. Since the static friction force provides the centripetal force of the turning, in order to avoid the centrifugal motion of the bicycle during the turning process and turning outward, the bicycle will turn outwards. Drive at the maximum safe speed.

$$\begin{gathered} \mu(m_1 + m_2)g \geqslant (m_1 + m_2)\frac{v_i^2}{R_c} \\ \Rightarrow v_i \leqslant \sqrt{\mu g R_c} \end{gathered} \tag{36}$$

Where, R_c refers to the radius of curvature of the curve at the inner point of the tangent. For each curve, there is always a unique value for the radius of curvature within the respective tangent. Therefore, the radius of curvature is always uniquely determined by the curve.

And because the less the speed is reduced, the more time it takes to enter the bend, so in order to minimize the time spent, the speed should be reduced as much as possible, but at the same time, it cannot exceed the maximum safe speed when the bicycle is turning. Therefore, we choose $\sqrt{\mu g R_w}$ as the speed at which the bike travels at the inner point of the curve:

$$v_i = \sqrt{\mu g R_w} \tag{37}$$

Similar to the previous discussion, the time elapsed from the entry point to the inside point of the cut into the curve is

$$\Delta t_4 = t(v_i) - t(v_1) \tag{38}$$

Where, v_i indicates the speed of the bike at the inner point of the curve.

The distance traveled by the bicycle in the Δt_4 time period is

$$x_1 = \int_{\Delta t_4} v_6 dt \tag{39}$$

Then, since the rider does not exert any force. Therefore, the energy consumed by the rider in this part of the process is

$$Q_{31} = 0 \tag{40}$$

Step2: The Inner Point of the Curve to the Exit Point

In the process from the inner point of the cut into the corner to the point of exit, we also assume that the force exerted by the rider is a constant force, then the tangential acceleration at this time can be expressed as:

$$a_6 = \frac{F_4 - f_1 - k_1 v_7}{m_1 + m_2} = \frac{dv_7}{dt} \tag{41}$$

Where, F_4 indicates the constant force applied by the rider during corner exit, v_7 indicates the speed of the bicycle from the inner point of the curve to the exit point.

Similar to the previous discussion, in the process of exiting the bend, the time expression of the velocity can be obtained:

$$v_7 = v_7(t) \tag{42}$$

Besides, the time elapsed from the inner point of the curve to the exit point is:

$$\Delta t_5 = t(v_i) - t(v_1) \tag{43}$$

The distance traveled by the bicycle in the Δt_5 time period is:

$$x_2 = \int_{\Delta t_5} v_7 dt \tag{44}$$

Besides, the energy consumed by the rider on the ramp is:

$$Q_{32} = F_4 x_2 \tag{45}$$

To simplify the notation, we record the turn time and the rider's energy expended as:

$$\begin{gathered} t_4 = \Delta t_4 + \Delta t_5 \\ Q_3 = Q_{31} + Q_{32} \end{gathered} \tag{46}$$

4.1.4 Optimization Model

Considering that there are three positional sections in the entire bicycle personal time trial, we summarize and integrate the above three positional sections to establish the objective function with the shortest competition time, the limit of the total energy consumed by the rider and the excess power from the past. The accumulation of curve accumulation is limited to the objective programming model of the constraints.

- **Objective Function**

In an individual time trial, the winner is the competitor who finishes the race in the least time. Therefore, we add up the time spent on the three position road segments, and take the shortest race time as the objective function.

$$min(t_a) = t_1 + t_2 + t_3(t_3') + t_4 \tag{47}$$

- **Restrictions**

Obviously, each different type of rider has its own limit value of total energy, and the accumulated energy consumed in the three-position sections of the individual time trial cannot exceed the limit value of total energy.

$$Q_1 + Q_2(Q_2') + Q_3 + Q_4 \leqslant Q_L \tag{48}$$

Where, Q_L indicates the limit value of the total energy consumed by the rider.

4.2 Solution of the Model

When solving the model, the search value method is used to obtain v_0, F_2, F_3 and F_1, v_b, v_b' then can be calculated. The ordinary differential equation is solved.

However, considering the actual situation, if only relying on conditional constraints to carry out the search value method, there are too many possibilities to traverse the search, the solution time is too long, or even the solution cannot be solved, which is not conducive to the solution.

Therefore, we should first consider the ability of all kinds of riders. First, we can obtain the general speed and strength of the rider by consulting the data, and then make the speed and strength fluctuate in the search, which narrows the search range and reduces the search process. degree of complexity.

Algorithm 1: Searching Method to Solve the Optimum Speed of Flat Road

Input: W_{max}, S, r_c, i

Output: Q, t

for i in range (11.1,13.9): **do**

Set step size=0.001

$a = \frac{dv}{dt}$、 corresponding inverse function : $t = t(v) + c, S = \int_0^t v(t)dt$, Combined with the given data, the bottom speed can be deduced. Use $Q < W_{max}$,Deduce the time required and the energy consumed Q, t.

if $Q < W_{max}$:

continue. The purpose is to make min(t)

Output: i **. This is the speed of the rider on smooth roads.**

end

5 Cyclist and Scene Model Applications

5.1 Define the Power Profiles of Two Types of Riders.

By consulting relevant literature and data preprocessing and cleaning, the power curves of male and female riders in the cycling schedule can be obtained.

5.1.1 Data Acquisition & Pre-processing

Manuel Mateo-March in the International Journal of Exercise Physiology and Performance documents the maximum power that male and female cyclists can maintain in a unit of time [4]. We carefully combed through the data recorded in the literature and data cleaning, and finally we got the maximum power data of male and female cyclists can maintain over different lengths of time. In addition, the corresponding websites are as follows:

Table2: Data source collation

Database Names	Database Websites Data	Type
MPMFC	https://journals.humankinetics.com/view/journals/ijspp/ijspp-overview.xml?tab_body=toc-10879	Data platform
CPDD	http://veloclinic.com/functional-pca-of-the%02golden-cheetah-power-duration-data/	Nutritional data
GCOD	https://github.com/GoldenCheetah/OpenData	Geography
Google Scholar	https://scholar.google.com/	Academic paper

5.1.2 Data Visualization

As mentioned in Document 4, the power-time curve of a cyclist is a decay curve. In an instantaneous time period, the maximum power output of male and female cyclists can be particularly large, reaching a power output of more than 1000 W, which is consistent with the actual power output. Consistent with the situation, for an instant period of time, the cyclist can be allowed to exert enough power to drive; for a longer duration period, the maximum power of male and female cyclists can only be maintained to a certain extent, and the degree is much lower than The maximum power in the instantaneous time period, because the longer the time, the more energy is consumed, the more metabolites such as lactic acid are accumulated, and the maximum power maintained by the rider is maintained at a lower level. The fitting result is shown in the following figure:

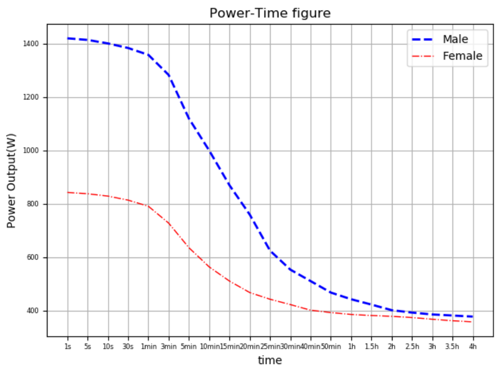


Figure 6: Power-Time figure

5.2 Apply the Model to Two Time Trials

Individual time trials are generally designed according to the situation of the organizer and combined with the actual scene. Among them, there are also tracks such as the time trial of the Tokyo 2021 Olympic Games in Japan. The 2021 UCI World Championship time trial is also taking place at the city circuit in Flanders, Belgium. According to the riding scenario, due to the large Elevation gain of the Mount Fuji circuit, we focused on the impact of uphill and downhill on runners. Since the UCI World Championships are mainly facing urban roads and there are many sharp turns, we focus on the impact of turns on the players. Image sourced from Google Maps: https://earth.google.com/web/

(a) Tokyo individual time trial

(b) UCI individual time trial

Figure 7: Real picture of the track

5.2.1 2021 Olympic Time Trial Course in Tokyo, Japan

● **Specific Introduction**

The time trial for the 2021 Tokyo Olympics in Japan will be held at the Mount Fuji circuit. Among them, one lap of the track is 22.1 kilometers. The distance for the women's race is one lap of the track and the distance for the men's race is two laps of the track. The men's track has an elevation gain of 846m and the women's track has an elevation gain of 423m.

(a) Tokyo Men's individual time trial

(b) Tokyo Women's individual time trial

(c) Rasterization Roadmap

Figure 8: Tokyo individual time trial

● **Model Analysis**

Consider that the women's race distance is one lap of the track, while the men's are twice as long as the women. Therefore, the above-mentioned rider position force application model is applied with a female rider as a reference object.

In order to get the length of the up and down slopes and the distance to the flat ground, we use rasterization to process the elevation-route relationship map. We focus on the cases of positions 1, 2, and 4.

● **Model Result**

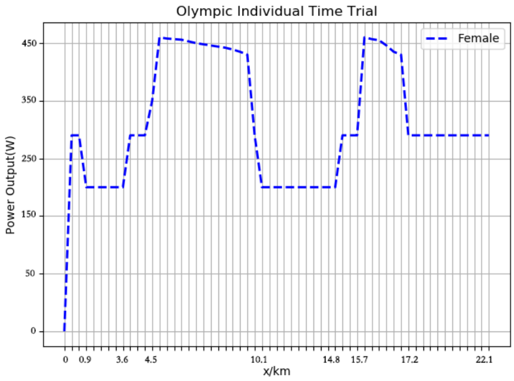


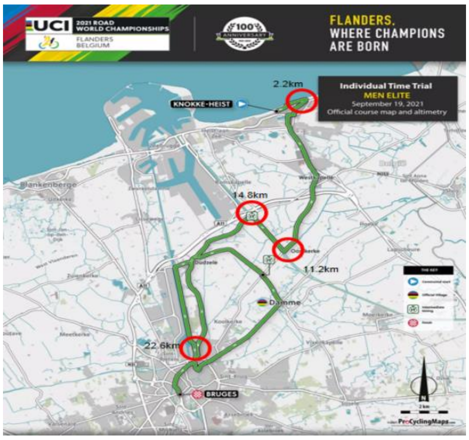
Figure 9: Power position curve diagram of female players

As shown in the figure above, the female cyclist runs at a constant power on the flat ground. The power increases briefly before going uphill, and the power decreases during the uphill process, but it still far exceeds the power on the flat ground, and maintains a lower power forward when going downhill.

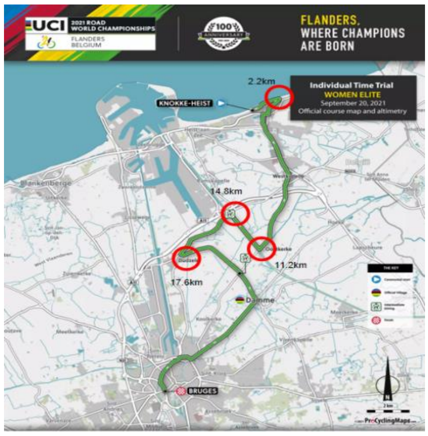
5.2.2 2021 UCI World Championship Time Trial Course in Flanders, Belgium

● **Specific Introduction**

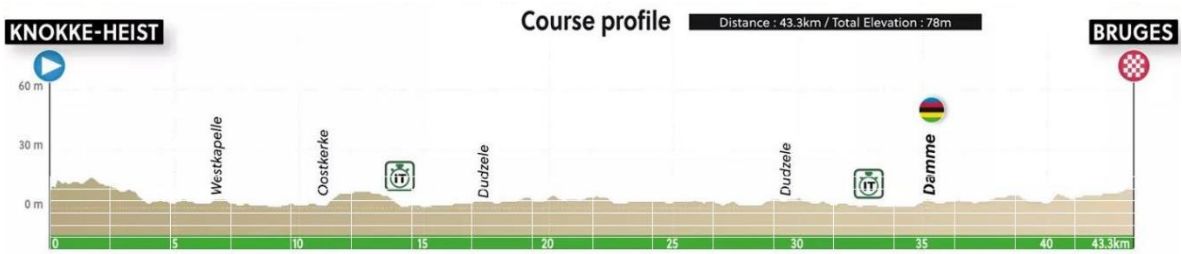
The 2021 UCI World Championship Time Trial takes place in Flanders, Belgium, on the North Sea beach. The entire track runs diagonally. The men's track has a total length of 43.30km and an altitude of 78m. The women's track has a total length of 30.30 kilometers and an elevation of 54 kilometers.



(a) Tokyo Men's individual time trial



(b) Tokyo Women's individual time trial



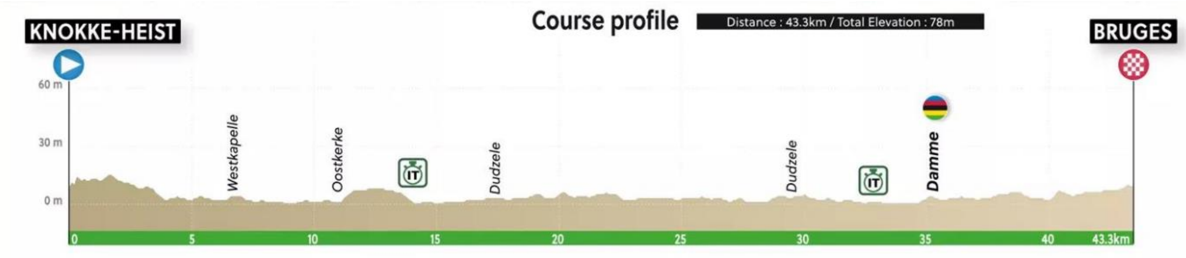


Figure 10: UCI individual time trial

● **Model Analysis**

For the track of the 2021 UCI World Championship time trial course in Flanders, Belgium. We take the driving situation of female cyclists as an example to study and use computer vision technology to vectorize the track route through pattern recognition, and use related models to train the route to predict the position of the sharp turn and the distance from the starting point. We focus on the cases of positions 1, 2, and 3.

● **Model Results**

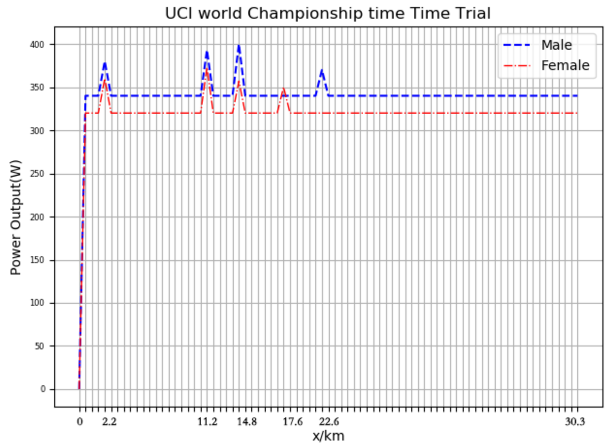


Figure 11: Power position curve diagram of players

As shown in the figure above, when the athlete faces a sharp turn, he first decelerates to the inner point of the cut bay, reduces the speed to the maximum speed allowed by the curve, and then quickly accelerates to the flat ground speed after passing the curve, and then continues on the flat ground. Drive evenly.

5.3 Design the Track

In the 1984 Los Angeles Olympic Games, the women's individual cycling event was officially included in the Olympic Games. When designing the track, we selected the track in Brand Park, California, hoping that the competitors can enjoy the joy of riding and recall the history of the personal bicycle time trial during the competition. We take Brand Park as the start and end of the track, the Elevation is 342m, the route is 26.37 kilometers long in one lap, and there are 4 sharp turns in total. The women's track is one lap of the route and the men's track is two laps of the route.

Figure 12: Designed track route map

- **Model Solution Results**

As shown in the Figure 13, since the designed track includes sharp turns, uphill and downhill, and flat ground, the model is comprehensively processed according to the above two situations.

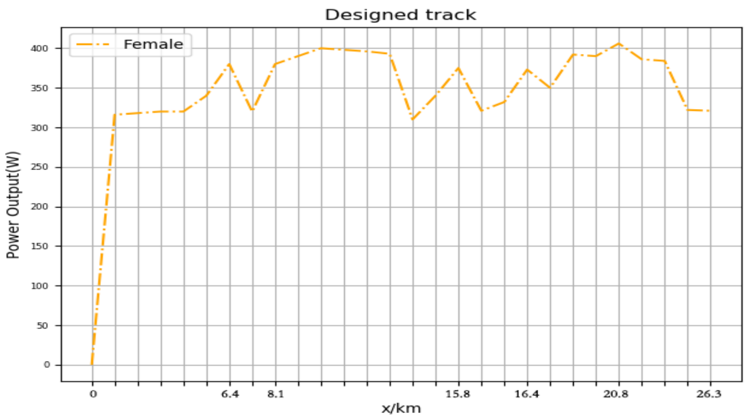


Figure 13: Designed track route map

6 Sensitivity Analysis

6.1 Sensitivity Analysis of Weather Conditions and Environment

Considering the potential impact of weather conditions [6], especially for wind direction and wind strength, race results may be better if the wind direction is the same as the direction the bike is traveling in and the wind is stronger; otherwise, the race results may be worse.

6.1.1 Wind Direction and Wind Strength Concept

In order to determine the influence of wind direction and wind strength on the model results, We take the 2021 Olympic Time Trial course as an example to analyze. When calculating the overall acceleration of the bicycle and the person at each position stage, the influence of the wind force is added.

$$a_n = a_b \pm \frac{F_w}{m_1 + m_2} \tag{49}$$

Where, a_n indicates the current acceleration of the bicycle and the person as a whole, a_b indicates the previous acceleration of the bicycle and the person as a whole, F_w indicates the wind strength.

When the positive sign in the above formula is taken, it means that the wind direction is the same as the driving direction, which is a facilitation effect; when the formula is negative, it means that the wind direction is opposite to the driving direction, which is a hindrance effect.

The formula for calculating wind is:

$$F_w = p \cdot s = 0.5 r_0 v^2 \cdot s \tag{50}$$

Where, p indicates wind pressure, s indicates cross-sectional area of an object, r_0 indicates air density, v indicates wind speed.

6.1.2 Sensitivity Analysis Results

We assume that under normal circumstances, r_0 is taken as 1.205kg/m^3, and the cross-sectional area of the cyclist is 0.36m^2. Therefore, in this sensitivity test, we take the total distance of 44.2km in the men's individual time trial and the cyclist's average race time of 60min as an example, take the wind speed of 4m/s in the breeze as the benchmark, and let it fluctuate by10% to explore the changes in the model results total game time and power-position curves. As follows:

Table 3: Total game time change results

Wind Direction	Wind Speed Change	Total Game Time Change
downwind	v+5%	-1.854%
	v-5%	+1.217%
headwind	v+5%	+1.148%
	v-5%	-1.516%

In the sensitivity test, when the wind direction is tailwind or headwind, the wind speed fluctuates up and down by 5%, and the total game duration do not change significantly. The model has passed the sensitivity test.

6.2 Sensitivity Analysis of Deviation from Target Power

Taking the 2021 Olympic Time Trial course in Tokyo, Japan as an example, we have derived an image of the cyclist's power-position curve, which represents the target power that the cyclist should apply at each particular section position, that is the optimal power. We only performed a sensitivity analysis on the image situation of female cyclists.

For the 2021 Olympic Time Trial course in Tokyo, Japan, our focus was on the rider's impact on downhill and downhill. Uphill versus downhill requires more traction and consumes more energy, making it harder to comply with our model's power plan. Therefore, randomly select a section of the uphill road, and the target power corresponding to the uphill position fluctuates randomly within the range of -1% to 1%, so as to determine the change of the completion time of the game. In addition, considering that the rider has some knowledge of the key parts of a given route and the possible range of its split time, it is assumed that the rider's time error at the top of the slope is not more than 1min.

Table 4: Total game time change results

Randomly Chosen Ramp	Power Fluctuation Range	Total Game Time Change
first uphill	-1%~1%	⩽1.236%
second uphill	-1%~1%	⩽1.317%

In the sensitivity test, when the uphill power fluctuates in the range of -1%~1%, the total game duration don't change significantly. The model has passed the sensitivity test.

7 Expansion of the model

In the road cycling team time trial, there are six runners on a team, and the time of the fourth team member crossing the finish line is taken as the team score. Therefore, according to the conventional analysis, two players are required to act as windbreakers, and the two windbreakers need to take turns to break the wind for the four sprinters at the front of the team, that is, to reduce the physical consumption of the sprinters, so that the sprinters can complete the long run. Distance traveled at higher speeds.

If the windbreaker is completely exhausted, the rest of the players should take turns to break the wind at the front of the team to reserve their strength for the rest of the team. Therefore, for the windbreaker, their race distance is not equal to the entire time trial of the team, but the distance that he can break the wind for the team.

To sum up, the dynamic model we established can be used for the windbreaker, so that it can maximize the benefits during the windbreaker.

8 Model Evaluation and Further Discussion

8.1 Strengths and Weakness

- **Strengths**

● In this model, we take into account different topographical conditions, and then carry out segmentation considerations, and the consideration of the dynamic model is more detailed. Taking shorter time as the goal and the athlete's physical fitness as the limit, the optimal power distribution scheme acceptable to the athlete was obtained.

● The dynamic model based on differential equation is scientific and reasonable.

● The model has good generality and scalability. If we need to consider the influence of other factors, we can modify the kinetic model to upgrade the model.

- **Weakness**

Due to insufficient data on athletes, the consideration of their tolerance may not be accurate enough, and the power allocation results may be biased.

8.2 Further Discussion

If there are more detailed data records of the athletes' low power recovery process after the overpower curve, then the use in the model can make the model results more accurate.

9 Conclusion

This paper analyzes in detail the movement of the rider on the gentle road, slope and curve of the individual time trial, and establishes the dynamic model of the rider's power-position. By studying the power-position curve of the rider, we propose that the rider drives with constant power on flat ground, increases power for a short time to reach the bottom of the slope, and then decelerates with constant traction to reach the top of the slope, during which the power drop decreases, Keep driving at lower power when going downhill. At the same time, on the curved road section, the rider first performs a natural deceleration motion to enter the corner and reaches the inner point of the cutting curve. The power during the cornering process is 0, and then exits the corner with constant traction, rapidly accelerates to the plane speed, and the power increases during the cornering process. Then continue to drive at a constant speed on the flat ground to keep the power constant.

In addition, in the sensitivity analysis of the model, we found that when the wind direction changes, the wind strength fluctuates up and down by 5%, and the target power for the uphill fluctuates randomly in the range of -1% to 1%, the change in the total duration of the game is not significant. Significantly. It means that the model has passed the sensitivity test.

Finally, to discuss the cycling team time trial with six riders, since the two windbreakers need to take turns breaking the wind for the four sprinters at the front of the team, the sprinters can complete long distances and high speeds. Therefore, we can maximize the benefit by using the established power-position dynamics model for the two windbreakers leading the entire team.

Rider's race guidance for a Directeur Sportif

I'm a bicycle road races researcher from MCM. As we all know, the distribution of power often plays a crucial role in the game. As the saying goes, sharpening the knife does not cut the woodwork by mistake, so the players attach great importance to the pre-match strategy formulation.

The physical fitness of a professional cyclist is much stronger than that of ordinary people, especially in lung capacity and heart rate. Lung capacity is extremely important for athletes in endurance sports like cycling, because the more oxygen you inhale, the more aerobic exercise you do, and the less anaerobic exercise, so the accumulation of lactic acid will increase. Lactic acid, it is known that the more lactic acid builds up, the easier it is to feel fatigued, and the muscles will have difficulty contracting. Secondly, in addition to high lung capacity, the heart rate of athletes is still very low, and the heart rate of professional cyclists is about 60% of that of ordinary people. To sum up, we need to develop a strategy to lower his heart rate and do as much aerobic exercise as possible, in other words, to complete the race at a lower power and make the time shorter.

From the perspective of riding posture, generally speaking, for road bicycles, there are mainly three contact points of hands, hips, and feet, and the corresponding contact points include handlebars, seat cushions, and pedals. The height, front and rear position, width, etc. will affect the correct riding posture of the bicycle, so a correct riding posture starts from the head and goes all the way to the feet. For the elbows, during the riding process, the elbows have a certain bending angle, and they are suspended in the air. Keep your shoulders relaxed during the ride, and don't let your shoulders stretch toward your ears.

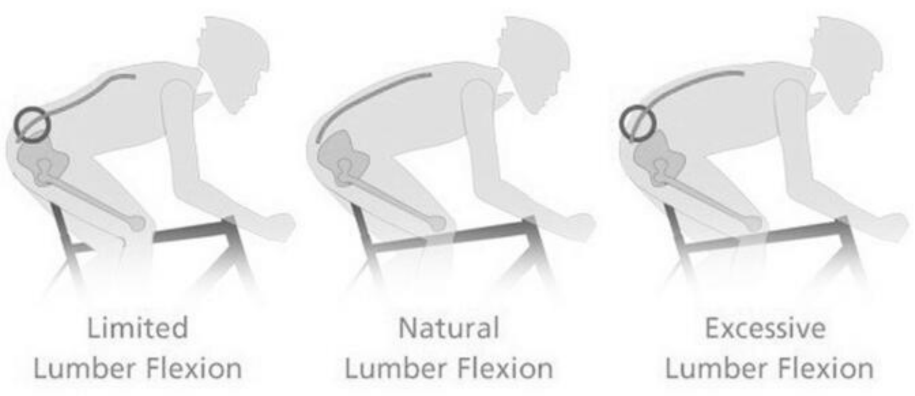


Figure from: https://news.cctv.com/2021/12/08/ARTILrDqi0F9MadXzEumJVGk211208.shtml

During the race, the terrain that needs special attention is slopes and sharp turns. We have formulated the following basic strategies: natural deceleration during sharp turns, rapid acceleration after sharp turns, early acceleration before going uphill, long and slow Downhill requires a traction force to make the downhill at a constant speed, and the steeper downhill will naturally accelerate downhill; at the same time, we aim to complete the race in a shorter time and limit the exercise load that athletes can bear, based on ordinary differential Equation, the dynamic optimization model is established. After solving the model by using the search value method, it is found that the power on the flat ground is moderate, and the acceleration process needs to be completed in a short time with a slightly larger power before going uphill. High power ensures that the speed reduction will not be too large after going uphill; on the contrary, when turning, you need to use a lower power to move at a constant speed when facing a gentle slope, but if it is a steep downhill, you do not need to apply traction, accelerate downhill naturally; in addition, when facing a sharp turn, you need to select a suitable entry point, so that the speed at the exit point is reduced to the maximum speed allowed by the sharp turn, after that, you need to use a slight Use large power to accelerate, and continue to drive at a constant speed with a moderate power when the speed increases to the flat ground speed.

As shown in the 2021 Olympic Time Trial in our model application and the conclusion of the 2021 UCI World Championships Time Trial in Flanders, Belgium.

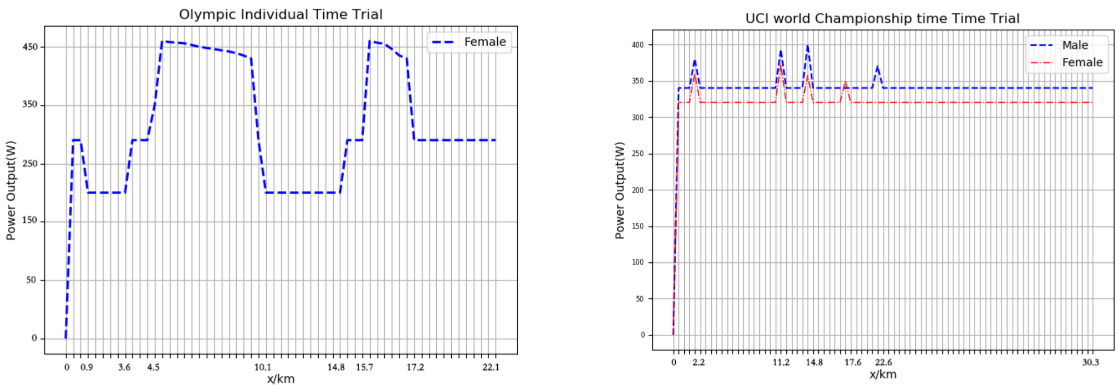


Figure: Power Position Dynamics Diagram

MCM
21th Feb 2022

10 References

[1] Wu Xinping. Discussion on pre-competition training of mountain bikes [C]//"Western Sports Research" 2013 Issue 1 (No. 129 in total). [Publisher unknown], 2013:78-80.

[2] Ding Danhua, Qiu Guomin.Mechanics of Bicycle Turning[J]. Discussion on Physics Teaching,2018,36(07):62-64.

[3] Zhang Mengcai. On the Rolling Friction in Cycling [J]. Technical Physics Teaching, 2005, 13(1): 22-22.

[4] [1] Zhang Yun. How to adjust the variable speed bicycle uphill to save effort [J]. Reference for Middle School Physics Teaching, 2012, 41(10): 40-41.

[5] Leo P, Spragg J, Podlogar T, et al. Power profiling and the power-duration relationship in cycling: a narrative review[J]. European journal of applied physiology, 2021: 1-16.

[6] Burnley M, Jones A M. Oxygen uptake kinetics as a determinant of sports performance[J]. European Journal of Sport Science, 2007, 7(2): 63-79.